

Credit Risk Characteristics of US Small Business Portfolios

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Motivation

- Focus in credit risk research has mainly been on modeling defaults of *corporate* firms.
- Small business remains underresearched.
- Modeling of joint defaults in standard models requires information on *market prices* and firm's *financials* (i.e. KMV, CreditMetrics).
- In *large* and *balanced* loan portfolios the main risk stems from many simultaneous default events.
- For determining tails of loss distributions, capturing *dependent defaults* is no less important than finding individual probabilities of default.

Objectives

- Analyse risk characteristics of a unique dataset on US retail loan portfolios.
- Richness of dataset allows introduction of multi-factor model of credit risk.
- Compare minimum capital requirements of Basel II with implication of our empirical results.
- Provide insights on financial crisis and credit risk.

Setting

Portfolio credit risk modeling in retail banking

- Concentration risk in a retail portfolio
 - ⇒ concentration does NOT come from a name concentration but from exposures to common correlated risk factors
- Lack of market prices
- Lack of financial statements
- Frequent default events
- Depends on probability of joint defaults

Small business

Small businesses in the US:

- Contribute about 50% to US GDP
- Employ 50% of the private workforce
- Loans to small businesses amount to 25% of commercial and industrial loans.

Small business *ambiguity*:

- Lack of market prices
- Lack of financial statements
- Standard credit risk models fail

Small business *information*:

- Frequent defaults on retail vs. relatively few defaults on corporate loans
- Internal risk ratings available at financial institutions for the Basel II capital requirements

Methodology (1/5)

Consider a portfolio of N firms. For $i \in 1, 2, \dots, N$ define $D_{i,t+1}$ to be a default indicator of company i , i.e. $D_{i,t+1} = 1$ if company defaults at time $t + 1$ and $D_{i,t+1} = 0$ otherwise.

Latent variable models

Default occurs if a latent variable $A_{i,t+1}$ (here: asset value of obligor i at time $t + 1$) falls below a default threshold L_i (liabilities).

Examples: Merton model (1974), CreditMetrics, KMV.

Simplification:

The basic message does not change if various exposures and loss given defaults are introduced.

Methodology (2/5)

Given a random vector $A_{t+1} = (A_{1,t+1}, \dots, A_{N,t+1})$ distributed with $\Phi(\cdot)$ and liability L_1, \dots, L_N , the default indicator is defined as:

$$D_{i,t+1} := \mathbb{1}_{\{A_{i,t+1} < L_i\}}$$

Default probability:

$$p_i := P[D_{i,t+1} = 1] = P[A_{i,t+1} < L_i] = \Phi(L_i)$$

Homogenous classes:

- Obligor are grouped together to form homogenous classes.
- The default threshold is chosen such that p_i equals average historical default frequency for firms with similar credit quality (as in CreditMetrics).

Methodology (3/5)

Multi-factor model:

- Assets of the obligor follow

$$A_{i,t} = w_k x_{k,t} + \sqrt{1 - w_k^2} \epsilon_{i,t}$$

where $A_{i,t}$ is obligor's i asset value,
 $x_{k,t} \sim N(0, \Omega)$ is a common factor,
 $\epsilon_{i,t} \sim N(0, 1)$ is a firm-specific factor,
and $E[x_{k,t} \epsilon_{i,t}] = 0$.

- Default threshold

$$A_{i,t+1} < \Phi^{-1}(\bar{p}_k) \iff D_{i,t+1} = 1$$

- Implied asset correlation

$$\rho_{kl} = w_k w_l \Omega_{kl}$$

Methodology (4/5)

Econometric methodology: GMM

- Joint probability of default: *theoretical*

$$p_{kl} = \int_{-\infty}^{\Phi^{-1}(\bar{p}_l)} \Phi \left(\frac{\Phi^{-1}(\bar{p}_k) - \Omega_{kl} w_k w_l y}{\sqrt{1 - \Omega_{kl}^2 w_k^2 w_l^2}} \right) \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} y^2 \right) dy$$

- Joint probability of default: *sample counterpart*

$$\hat{p}_{kl} = \frac{1}{T} \sum_{t=1}^T (p_{k,t+1} \cdot p_{l,t+1})$$

where $p_{k,t+1}$ is the observed default frequency (ODF) of obligor's class k .

Methodology (5/5)

Joint default events

- Joint probability of default p_{ij} for obligor i and j :

$$\hat{p}_{ij} = \frac{1}{T} \sum_{t=1}^T D_{i,t+1} D_{j,t+1}$$

- Joint probability of default: *sample counterpart*

$$\begin{aligned}\hat{p}_{kl} &= \frac{1}{N_{k,t} N_{l,t}} \sum_{i \in k}^{N_{k,t}} \sum_{j \in l}^{N_{l,t}} \frac{1}{T} \sum_{t=1}^T (D_{i,t+1} \cdot D_{j,t+1}) \\ \hat{p}_{kl} &= \frac{1}{T} \sum_{t=1}^T \frac{\sum_{i \in k}^{N_{k,t}} D_{i,t+1}}{N_{k,t}} \frac{\sum_{j \in l}^{N_{l,t}} D_{j,t+1}}{N_{l,t}} \\ \Rightarrow \hat{p}_{kl} &= \frac{1}{T} \sum_{t=1}^T (p_{k,t+1} \cdot p_{l,t+1})\end{aligned}$$

Data

- Nearly 240,000 US small businesses in Dun & Bradstreet's proprietary database
- Firms' actual borrowing and payment behavior
- Quarterly information from calendar years 2005 through 2011
- Credit rating on a scale of 1–10
- 10 industry sectors

Data

Table I Small businesses in the US

		# firms	% total	min	max	defaults (%)
1. Industry	Agriculture, Forestry, Fishing (A)	9,902	4.19	9,340	10,188	8.39
	Mining (B)	825	0.35	758	872	12.55
	Construction (C)	32,180	13.61	27,048	36,275	13.13
	Manufacturing (D)	16,382	6.93	14,155	18,278	17.48
	Transportation, Communications, Electric, Gas, and Sanitary Services (E)	8,123	3.44	6,963	9,046	14.12
	Wholesale Trade (F)	16,048	6.79	14,063	17,836	16.02
	Retail Trade (G)	35,032	14.82	29,552	39,993	14.19
	Finance, Insurance, and Real Estate (H)	20,020	8.47	17,170	22,310	11.34
	Services (I)	96,379	40.78	85,672	104,065	11.19
	Public Administration and non-classified (J)	1,467	0.62	1,358	1,831	23.88
2. Firm size	1-5	133,755	56.59	115,434	147,547	9.67
	6-10	44,125	18.67	38,308	49,158	12.89
	11-20	28,244	11.95	24,731	31,174	15.82
	21-30	10,890	4.61	9,778	11,867	18.53
	31-50	9,150	3.87	8,344	9,904	21.16
	51-100	6,149	2.60	5,670	6,751	26.42
	>100	4,043	1.71	3,700	4,446	35.98
3. \$ outstanding	\$0-500	38,530	16.30	29,436	48,676	5.78
	\$501-1,000	18,648	7.89	15,510	24,119	7.57
	\$1,001-2,000	22,880	9.68	19,990	27,531	9.15
	\$2,001-5,000	32,174	13.61	29,208	35,538	10.94
	\$5,000-15,000	48,536	20.54	42,366	52,458	12.67
	\$15,001-30,000	28,001	11.85	24,930	31,288	14.73
	>\$30,001	47,589	20.13	38,951	53,303	22.23
4. Region	Central	17,512	7.41	16,135	18,876	10.65
	West	53,754	22.74	45,590	59,743	12.84
	Northeast	49,437	20.92	43,212	54,240	12.37
	Midwest	36,319	15.37	32,368	39,741	12.31
	Southeast	55,219	23.36	47,174	61,552	14.00
	Southwest	24,118	10.20	21,533	26,437	12.62
5. Private	Yes	236,284	99.97	206,140	260,471	12.74
	No	74	0.03	50	117	42.58
Total		236,358	100.00	206,196	260,590	12.74

Data

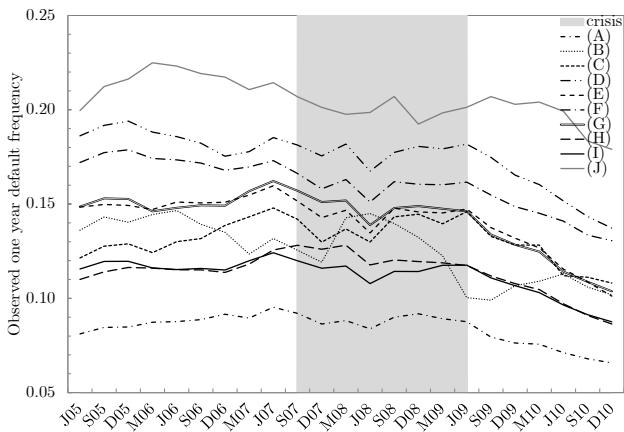


Figure 1: Observed default frequencies per industry classes

Results:

Which risk dominates in small businesses: systematic or idiosyncratic?

Table III Sensitivity to the credit rating & industry common risk factors

Industry	Credit rating		Sensitivity w_k (%)							
	1	2	3	4	5	6	7	8	9	10
<i>Panel A: w_k to the credit rating & industry common risk factors</i>										
(A) Agriculture	3.70 (4.05)	0.00 (3.73)	5.54 (3.96)	0.00 (3.9)	7.74* (4.27)	8.39** (4.04)	7.61* (4.53)	6.86 (4.52)	6.34 (4.54)	6.14 (4.80)
(B) Mining	0.00 (7.81)	18.41 (12.86)	2.76 (11.23)	0.00 (11.87)	6.58 (13.11)	0.00 (12.78)	0.00 (13.53)	0.00 (15.53)	0.00 (15.59)	16.07 (18.79)
(C) Construction	5.49* (2.92)	5.59** (2.23)	4.84** (2.16)	5.00** (2.2)	6.44*** (1.79)	5.94*** (1.83)	6.14*** (1.71)	6.37*** (1.61)	7.82*** (1.47)	8.72*** (1.53)
(D) Manufacturing	5.85** (2.76)	4.46 (3.07)	5.43** (2.76)	6.15** (2.6)	4.59 (2.84)	4.68* (2.7)	6.69** (2.7)	6.10** (2.37)	7.06*** (2.25)	6.31** (2.64)
(E) Transportation	0.00 (4.21)	2.33 (4.67)	6.52 (4.57)	8.11* (4.15)	0.00 (3.94)	5.61 (4.04)	5.15 (3.88)	7.41** (3.51)	7.78** (3.82)	8.97*** (3.36)
(F) Wholesale	4.34 (2.7)	6.45** (2.71)	2.19 (2.66)	5.74* (2.98)	6.43** (2.77)	2.57 (2.78)	6.49** (2.73)	4.61 (2.87)	7.17*** (2.54)	6.29** (3.00)
(G) Retail	3.81 (2.62)	6.71*** (2.33)	6.30*** (1.95)	7.10*** (1.69)	7.70*** (1.67)	6.77*** (1.67)	7.14*** (1.57)	6.37*** (1.41)	6.59*** (1.33)	8.85*** (1.45)
(H) Finance	3.77 (2.67)	4.75* (2.73)	2.62 (2.47)	6.72*** (2.22)	4.63* (2.73)	4.94* (2.73)	8.68*** (2.43)	6.92*** (2.52)	7.63*** (2.89)	7.58** (3.01)
(I) Services	3.68*** (1.23)	3.68*** (1.17)	4.19*** (1.03)	5.03*** (1.01)	4.99*** (0.99)	4.96*** (0.94)	5.40*** (0.91)	5.40*** (0.93)	5.68*** (0.94)	7.22*** (1.00)
(J) non-classified	7.36 (4.77)	0.00 (5.65)	0.00 (6.59)	5.77 (7.77)	5.56 (8.59)	5.93 (8.58)	11.83 (8.39)	12.00 (8.68)	6.18 (8.45)	9.69 (7.46)

Results:

Considerable shifts in the provisions level and in the spread of loss distribution

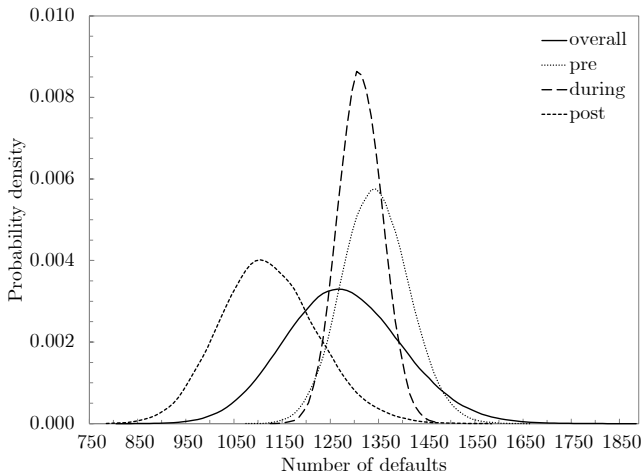


Figure 4: Portfolio loss distribution comparison

Results:

Capital requirements for corporate exposures implied by our estimates of asset correlation are in line with the regulatory ones

Table VIII Capital requirement for corporate debt in the US

Panel A: Capital requirement for corporate portfolios

	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
$\bar{\rho}$ (%)	0.19	0.17	0.13	0.16	0.16	0.16	0.24	0.30
ρ_{ii} (%)	20.72 (6.33)	20.28 (6.31)	19.34 (6.40)	11.57 (4.60)	20.11 (6.46)	11.54 (4.40)	9.57 (3.78)	13.20 (4.60)
K_R (%)	4.14	3.92	3.45	3.83	3.83	3.79	4.72	5.31
K_M (%)	3.64 (1.25)	3.33 (1.17)	2.73 (1.00)	1.69 (0.72)	3.22 (1.16)	1.66 (0.68)	1.77 (0.73)	2.89 (1.09)
Difference (%)	0.50	0.58	0.72	2.15	0.61	2.13	2.95	2.42
t -statistic	0.40	0.50	0.71	2.99***	0.53	3.13***	4.06***	2.23**

	BB+	BB	BB-	B+	B	B-	CCC/C
$\bar{\rho}$ (%)	0.68	0.44	0.47	1.61	2.92	6.24	23.97
ρ_{ii} (%)	13.80 (4.50)	11.31 (4.05)	3.83 (1.86)	15.02 (4.52)	16.81 (5.14)	22.18 (5.45)	15.92 (6.00)
K_R (%)	7.59	6.30	6.55	10.16	11.88	14.92	22.24
K_M (%)	4.93 (1.66)	3.08 (1.14)	1.20 (0.50)	8.82 (2.56)	13.33 (3.71)	23.38 (4.72)	25.81 (5.09)
Difference (%)	2.66	3.23	5.35	1.34	-1.45	-8.47	-3.57
t -statistic	1.60	2.82***	10.67***	0.52	0.39	1.79*	0.70

Panel B: Paired difference test ($H_0 : \bar{K}_R - \bar{K}_M = 0$)

	LGD	M	Mean (%)	SD (%)	t	df	p -value
Pair $K_R - K_M$	0.5	3	0.74	3.28	0.88	14	0.39
	0.9	3	1.34	5.92	0.87	14	0.40
	0.5	14	3.17	7.04	1.74	14	0.10

Results:

Small businesses are subject to inefficient capital allocation

Table IX Capital requirement for retail portfolios

		Credit rating: 1 2 3 4 5 6 7 8 9 10									
(A)	\bar{K}_R	6.01	5.98	5.96	6.04	6.09	6.33	6.71	6.97	8.21	9.48
	K_m	0.73	0.00	1.07	0.00	1.78	2.27	2.35	2.25	2.64	3.09
		(0.91)	(0.80)	(0.88)	(0.90)	(1.13)	(1.25)	(1.57)	(1.65)	(2.05)	(2.56)
	κ_r/κ_m	8.23	NA	5.55	NA	3.43	2.79	2.86	3.10	3.11	3.07
(C)	\bar{K}_R	6.44	6.20	6.33	6.48	6.61	6.90	7.22	7.78	9.00	10.28
	K_m	1.48	1.34	1.23	1.36	1.89	1.88	2.11	2.46	3.73	5.00
		(0.85)	(0.58)	(0.59)	(0.65)	(0.58)	(0.63)	(0.64)	(0.67)	(0.75)	(0.92)
	κ_r/κ_m	4.35	4.64	5.16	4.77	3.51	3.67	3.42	3.16	2.41	2.06
(D)	\bar{K}_R	7.50	7.32	7.40	7.59	7.73	8.00	8.40	8.83	9.57	10.39
	K_m	2.13	1.53	1.92	2.29	1.71	1.84	2.88	2.79	3.62	3.63
		(1.07)	(1.13)	(1.05)	(1.05)	(1.13)	(1.12)	(1.24)	(1.14)	(1.21)	(1.55)
	κ_r/κ_m	3.52	4.79	3.85	3.32	4.51	4.35	2.92	3.17	2.64	2.86
(E)	\bar{K}_R	6.78	6.86	6.87	6.89	6.95	7.09	7.49	7.88	8.54	9.82
	K_m	0.00	0.69	2.07	2.66	0.00	1.86	1.85	2.96	3.46	4.83
		(1.40)	(1.61)	(1.62)	(1.52)	(1.35)	(1.47)	(1.52)	(1.51)	(1.82)	(1.90)
	κ_r/κ_m	NA	9.98	3.32	2.60	NA	3.81	4.05	2.66	2.46	2.03
(F)	\bar{K}_R	7.43	7.15	7.35	7.61	7.67	7.82	8.21	8.63	9.40	10.24
	K_m	1.52	2.19	0.73	2.13	2.44	0.95	2.71	2.01	3.59	3.53
		(1.00)	(1.00)	(0.96)	(1.19)	(1.13)	(1.10)	(1.22)	(1.31)	(1.34)	(1.73)
	κ_r/κ_m	4.88	3.26	10.06	3.57	3.15	8.24	3.03	4.30	2.62	2.90
(G)	\bar{K}_R	6.86	6.68	6.83	6.96	6.97	7.22	7.43	7.79	8.57	9.78
	K_m	1.15	2.02	1.97	2.33	2.56	2.35	2.61	2.47	2.91	4.73
		(0.84)	(0.77)	(0.67)	(0.61)	(0.62)	(0.64)	(0.63)	(0.59)	(0.63)	(0.82)
	κ_r/κ_m	5.95	3.31	3.47	2.99	2.73	3.07	2.85	3.16	2.95	2.07
(H)	\bar{K}_R	6.56	6.33	6.49	6.55	6.60	6.85	7.08	7.74	8.61	9.77
	K_m	1.03	1.20	0.69	1.93	1.31	1.52	3.01	2.67	3.43	4.02
		(0.78)	(0.75)	(0.70)	(0.70)	(0.83)	(0.90)	(0.94)	(1.06)	(1.39)	(1.67)
	κ_r/κ_m	6.35	5.26	9.45	3.39	5.04	4.51	2.36	2.90	2.51	2.43
(I)	\bar{K}_R	6.31	6.25	6.35	6.50	6.60	6.78	7.07	7.48	8.40	9.66
	K_m	0.90	0.87	1.06	1.38	1.42	1.49	1.78	1.94	2.42	3.75
		(0.32)	(0.30)	(0.28)	(0.30)	(0.31)	(0.31)	(0.32)	(0.36)	(0.43)	(0.55)
	κ_r/κ_m	7.00	7.15	6.00	4.71	4.65	4.53	3.98	3.85	3.47	2.57

Results for (B) and (J) available in the paper

Robustness:

Parameter uncertainty – assumption about normality of common risk factors

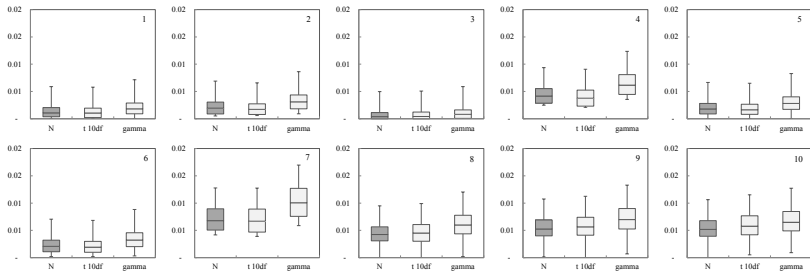


Figure 5: Parameter uncertainty

Conclusions

- Sensitivity to obligor class-specific common factors remains low for small business.
- Small businesses have a tendency to depend mainly on *idiosyncratic risk* (96-100%).
- During the crisis firms which withstood the deterioration of macroeconomic conditions did not go systematically into default. Due to their higher resistance to changes in the common risk factors the *correlation was low*. Lower than corporates.
- Basel II *overestimates* the required capital for retail portfolios. Relatively safe investment.
- The *more creditworthy* small firms suffer the highest capital charges relatively to their riskiness.

Thank you for your attention!

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