Information Noise and Bankruptcy Forecasting

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* The paper was written at UNSW, Australia. The views expressed in this paper are those of the author and do not necessarily reflect those of the employer.

Motivation

• Real-world information is noisy and imperfect for modelling.

• In credit risk research and practices, such concerns are prevalent. They become more important, particularly during/post GFC
  – Are we comparing like-for-like if not controlling for noise in the information used to assess (single-name) default risk?

• This topic received little attention in the academic literature on empirical credit risk. However, conflicting findings exist, likely due to information noise

• Empirical models accounting for noise in the input data are highly desirable, but are virtually non-existent
Paper In A Nutshell

• Current practice in bankruptcy/default prediction

\[ \log(\lambda_t) = \beta'X_t \quad \text{or} \quad \text{logit}(\lambda_t) = \beta'X_t \]

where \( \lambda_t \) is hazard rate, \( X_t \) is time-varying covariates, \( \beta \) is coefficients

• When there is information noise in the input data, a theory (Duffie and Lando [2001], DL) implies the Right-Hand-Side is nonlinear

• We propose new hazard specifications to approximate this nonlinearity. They also construct an empirical test on the DL theory

• We demonstrate strong empirical evidence supporting hypotheses regarding our specifications, including
  – Full-sample test on the signs of coefficients and Goodness-of-Fit
  – Out-of-sample test results on forecasting accuracy
  – Robustness checks
Theory – Assumptions

• There is a noise in the accounting reports of a debt issuer (firm), which is
  – Associated with log(asset)
  – Normally distributed, $\sim \mathcal{N}(u, a^2)$ *

• Only noisy value of assets is observed by creditors/modellers

• Firm files bankruptcy (or defaults) when true log(asset) first falls below a low boundary, $v$

• $a$ is “a measure of the degree of noise” (DL)

* We assume unbiased accounting reports ($u = -\frac{a^2}{2}$) throughout the paper
Theory – Results

• A filtering problem: Conditional on the observed (noisy) asset value, what is the conditional Probability of Bankruptcy (PB)?

• DL provided an analytical solution to this problem

• What does the theoretical PB look like?
The plot when the observed (noisy) asset growth rate is 0

Source: Figure 4 in Duffie and Lando [2001, Econometrica], with permissions from Econometrica and the authors
The higher degree of noise, the less responsive of observed asset growth rate (return) to PB, and vice versa
Projection on $PB - r_N$
log $PB - \alpha - r_N$

The pattern persists for any monotonic transformation, e.g. log or logit, of PB
Projection on log $PB - r_N$

Similar patterns exist for other covariates, e.g. leverage, expected asset return, volatility (see Appendix)
Our Approach – 1

• **Step 0**: PB can be approximated by hazard rate,

\[ \lambda_t = \lim_{\Delta t \to 0} \frac{PB(t, t + \Delta t)}{\Delta t} \]

The limit exists when there is incomplete information (see DL). Here we use Cox [1972] Proportional Hazard Model to model \( \lambda_t \).

• **Step 1**: Identify time-varying covariates, \( X_t \)

We choose covariates, \( X_t \), from 4 well-known, well-accepted models (**reference models**) in the literature

- Shumway [2001, JoB] (S01 Model)
- Chava and Jarrow [2004, RoF] (CJ04 Model)
- Simplified version of Duffie, Saita and Wang [2007, JFE] (DSW07-S Model)
- Bharath and Shumway [2008, RFS] (BS08 Model)
Our Approach – 2

• **Step 2:** Augment the data with a proxy for degree of noise, $\tilde{a}$, such that $\tilde{a}$ is decreasing in $a$

We choose well-accepted proxies from the Finance literature

– **Firm size** (log[Total Assets] in main results, log[Equity], [Asset Rank] in robustness checks)

– **Analyst coverage & Analysts’ forecast variation** (in robustness checks)

• **Step 3:** Create new models (augmented models) by adding interaction effects to hazard function

$$\log(\lambda_t) = \bar{\beta}'X_t + \bar{\gamma}_0\tilde{a} + \sum_{i=1}^{I} \bar{\gamma}_i(\tilde{a} \ast X^i)$$

where $\bar{\beta}, \bar{\gamma}$ are coefficients on main and interaction effects
Three Testable Hypotheses – 1

• **Hypothesis 1**: We can explicitly predict the sign of coefficients on interaction effects, $\bar{\gamma}_i$

$$\log(\lambda_t) = \bar{\beta}' X_t + \bar{\gamma}_0 \bar{a} + \sum_{i=1}^{I} \bar{\gamma}_i (\bar{a} \ast X^i)$$

  - If $X^i$ is increasing in $\lambda_t$, then $\bar{\gamma}_i > 0$
  - If $X^i$ is decreasing in $\lambda_t$, then $\bar{\gamma}_i < 0$

• **Example**:

  - **Observed (noisy) asset return** is **decreasing** in $\lambda_t$
  - When interacting it with **firm size** (a proxy that is decreasing in $\alpha$), the interaction effect should have a **negative** coefficient
Three Testable Hypotheses – 2

• **Hypothesis 2**: In-sample model fit is improved for augmented models, compared to reference models

• **Hypothesis 3**: Out-of-sample forecasting accuracy is improved in augmented models, compared to reference models

Rationale: if DL theory reflects the reality, then our specifications better approximate the true Data Generation Process, resulting better model performance
Data

Our Bankruptcy Panel Dataset (1979-2012)
2.15M firm-months, 2,112 bankruptcies
from a total of 20,180 North American public firms

Bankruptcy (Response Variables)
- New Generation www.BankruptcyData.com
- UCLA-LoPucki
- Mergent FISD
- Compustat deletion reason “2”

Independent Variables
- IBES (analyst forecasts)
- Datastream (3m T-rate)
- Compustat Quarterly / Annual
- CRSP Monthly
Data Summary Statistics (see the paper)
Empirical Results

• For all firms (see appendix) and non-financial firms (see the paper)

• General conclusions
  – The data is consistent with Hypothesis 1, regarding the signs of coefficients on the interaction effects
  – The data strongly supports Hypothesis 2: in-sample model fit improves significantly
  – Strong evidence to support Hypothesis 3: out-of-sample forecasting accuracy improves significantly
    By two predictability measures: Area-Under-Curve & captured bankruptcies within deciles
  – Results are robust to alternative empirical choices
Contributions

• First attempts to empirically test and explore implications of the theory of credit risk modelling with incomplete information (DL & sequel)

To our best knowledge, no empirical study has explored this strand of literature

• Our specifications reconcile conflicting empirical findings in the literature of corporate bankruptcy/default prediction
  – Firm size (Shumway [2001], Duffie et al. [2007], Campbell et al. [2011, JOIM])
  – NI/TA (Bharath & Shumway [2008], Chava et al. [2011, Management Science])

• New control variables for credit risk-related empirical studies, addressing noise in econometricians’ information set

• New hazard specifications for industry credit rating practices
  – They address information noise in the input data for PD modelling
  – We demonstrate that the effects generally exist in reality

• We provide a mechanism to elegantly handle outliers
Future Work

This paper opens a number of opportunities for future empirical research

• Default prediction, private firms (bank loans)
• Impact of information bias, in addition to noise
• Applications of the method on assessing the proxies for intensity of information asymmetry
  – Market liquidity
  – Corporate governance
• Other ways to account for information quality
Appendix
Empirical Results – 1

• Full-sample Estimates*: Hypothesis 1, regarding the signs of coefficients on the interaction effects

• The data is highly consistent with the prediction of Hypothesis 1
  – Results using Chava and Jarrow [2004] (CJ04 Model) as the reference model (similar results for other models, see the paper)

* Robust standard errors, corrected for heteroskedasticity, are adopted. There is no need to adjust for firm-level clustering, as Cox model takes each firm as unit of analysis. We only take the first bankruptcy for each firm if applicable.
## Full-sample Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>CJ04 Model</th>
<th>Augmented CJ04 Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NI/TA</strong></td>
<td>-0.38</td>
<td>-1.57*** (0.38)</td>
</tr>
<tr>
<td><strong>TL/TA</strong></td>
<td>2.85*** (0.13)</td>
<td>2.09*** (0.19)</td>
</tr>
<tr>
<td><strong>EXRET</strong></td>
<td>-1.64*** (0.13)</td>
<td>-0.14 (0.10)</td>
</tr>
<tr>
<td><strong>RSIZE</strong></td>
<td>-0.18*** (0.02)</td>
<td>-0.35*** (0.02)</td>
</tr>
<tr>
<td><strong>σₑ</strong></td>
<td>0.20*** (0.02)</td>
<td>0.12*** (0.04)</td>
</tr>
<tr>
<td><strong>IND2</strong></td>
<td>-0.50*** (0.14)</td>
<td>-0.53*** (0.15)</td>
</tr>
<tr>
<td><strong>IND3</strong></td>
<td>-0.14 (0.26)</td>
<td>0.22 (0.26)</td>
</tr>
<tr>
<td><strong>IND4</strong></td>
<td>-0.64 (0.49)</td>
<td>-0.24 (0.47)</td>
</tr>
<tr>
<td><strong>NI/TA*IND2</strong></td>
<td>0.89** (0.39)</td>
<td>0.77** (0.37)</td>
</tr>
<tr>
<td><strong>TL/TA*IND2</strong></td>
<td>0.51*** (0.18)</td>
<td>0.56*** (0.18)</td>
</tr>
<tr>
<td><strong>NI/TA*IND3</strong></td>
<td>0.13 (0.60)</td>
<td>0.69 (0.58)</td>
</tr>
<tr>
<td><strong>TL/TA*IND3</strong></td>
<td>0.29 (0.30)</td>
<td>-0.39 (0.30)</td>
</tr>
<tr>
<td><strong>NI/TA*IND4</strong></td>
<td>-3.30*** (0.72)</td>
<td>-2.40*** (0.78)</td>
</tr>
<tr>
<td><strong>TL/TA*IND4</strong></td>
<td>-0.24 (0.58)</td>
<td>-1.13** (0.54)</td>
</tr>
<tr>
<td><strong>log(TA)</strong></td>
<td>-0.04 (0.04)</td>
<td>0.00 (0.04)</td>
</tr>
<tr>
<td><strong>EXRET*log(TA)</strong></td>
<td>-0.30*** (0.02)</td>
<td>0.00 (0.02)</td>
</tr>
<tr>
<td><strong>NI/TA*log(TA)</strong></td>
<td>-0.26*** (0.08)</td>
<td>0.00 (0.02)</td>
</tr>
<tr>
<td><strong>σₑ*log(TA)</strong></td>
<td>0.04** (0.02)</td>
<td>0.00 (0.02)</td>
</tr>
<tr>
<td><strong>TL/TA*log(TA)</strong></td>
<td>0.15*** (0.04)</td>
<td>0.00 (0.02)</td>
</tr>
</tbody>
</table>
Empirical Results – 2

• Likelihood Ratio Tests: Hypothesis 2, regarding the in-sample model fit

• The data strongly supports Hypothesis 2: (log)Likelihood increases significantly
In-sample Goodness-of-Fit

Likelihood ratio test

- Constrained model: reference model
- Unconstrained model: augmented model
- Under the Null Hypothesis, “difference of -2logL” ($\chi^2$-statistics) is $\chi^2$-distributed with d.f. as (number of additional variables)

Alternative Goodness-of-Fit measures (McFadden's pseudo-$R^2$, AIC) provide similar results
Empirical Results – 3

• Out-of-Sample Tests: Hypothesis 3
  – 10 holdout samples, 2003–2012 (very similar results with bootstrapped 1,000 holdout samples)

• Measure 1: Area Under ROC Curve (AUC)

  The augmented models significantly improve AUC in typically 6 out of 10 holdout years, but are never significantly worse
## Out-of-Sample AUC (CJ04)

<table>
<thead>
<tr>
<th>Holdout Period</th>
<th># of Firms</th>
<th># of Bankruptcy</th>
<th>CJ04 Model (3)</th>
<th>Augmented CJ04 Model (4)</th>
<th>AUC Difference (4)-(3)</th>
<th>$(\chi^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>6172</td>
<td>101</td>
<td>0.9244</td>
<td>0.9350</td>
<td>$1.06 \times 10^{-2}$</td>
<td><strong>(4.92)</strong></td>
</tr>
<tr>
<td>2004</td>
<td>5845</td>
<td>52</td>
<td>0.9218</td>
<td>0.9391</td>
<td>$1.73 \times 10^{-2}$</td>
<td>***/(8.56)</td>
</tr>
<tr>
<td>2005</td>
<td>5593</td>
<td>40</td>
<td>0.9340</td>
<td>0.9426</td>
<td>$8.60 \times 10^{-3}$</td>
<td>***/(14.09)</td>
</tr>
<tr>
<td>2006</td>
<td>5531</td>
<td>30</td>
<td>0.8884</td>
<td>0.9162</td>
<td>$2.78 \times 10^{-2}$</td>
<td>**/(4.07)</td>
</tr>
<tr>
<td>2007</td>
<td>5471</td>
<td>26</td>
<td>0.8993</td>
<td>0.8907</td>
<td>$-8.60 \times 10^{-3}$</td>
<td>1.28</td>
</tr>
<tr>
<td>2008</td>
<td>5275</td>
<td>61</td>
<td>0.8882</td>
<td>0.9051</td>
<td>$1.69 \times 10^{-2}$</td>
<td>***/(8.81)</td>
</tr>
<tr>
<td>2009</td>
<td>5150</td>
<td>122</td>
<td>0.8734</td>
<td>0.8791</td>
<td>$5.70 \times 10^{-3}$</td>
<td>0.85</td>
</tr>
<tr>
<td>2010</td>
<td>4839</td>
<td>43</td>
<td>0.9292</td>
<td>0.9387</td>
<td>$9.50 \times 10^{-3}$</td>
<td>*(3.47)</td>
</tr>
<tr>
<td>2011</td>
<td>4704</td>
<td>37</td>
<td>0.9019</td>
<td>0.9139</td>
<td>$1.20 \times 10^{-2}$</td>
<td>1.41</td>
</tr>
<tr>
<td>2012</td>
<td>5056</td>
<td>46</td>
<td>0.9091</td>
<td>0.9215</td>
<td>$1.24 \times 10^{-2}$</td>
<td>2.25</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td>0.9070</td>
<td>0.9182</td>
<td>$1.12 \times 10^{-2}$</td>
<td><strong>/(21.79)</strong></td>
</tr>
</tbody>
</table>

- E.g., CJ04 Model (see other results in the paper)
- Positive (negative) AUC difference means better (worse) performance
- Test on correlated ROC (Delong et. al) are used. *** different from zero at 1% significance level, ** at 5% level, * at 10% level
Empirical Results – 3 (Cont’d)

• Measure 2: Captured fractions of the total number of bankruptcies within deciles ranked by model forecasts

More bankruptcies captured in top deciles (better classification)

Less captured in lowest deciles (less mis-classification)

Higher cumulative captured bankruptcies in all deciles
Out-of-Sample Captured B’tcy by Deciles

Panel A: Fractions of bankruptcies captured within deciles ranked by model forecasts (%)

<table>
<thead>
<tr>
<th>Decile</th>
<th>S01 Model</th>
<th>Augmented S01 Model</th>
<th>CJ04 Model</th>
<th>Augmented CJ04 Model</th>
<th>DSW07-S Model</th>
<th>Augmented DSW07-S Model</th>
<th>BS08 Model</th>
<th>Augmented BS08 Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74.01</td>
<td>74.91</td>
<td>75.63</td>
<td>76.34</td>
<td>71.15</td>
<td>72.22</td>
<td>74.73</td>
<td>75.45</td>
</tr>
<tr>
<td>3</td>
<td>6.09</td>
<td>5.56</td>
<td>5.20</td>
<td>5.02</td>
<td>4.66</td>
<td>5.02</td>
<td>4.12</td>
<td>4.30</td>
</tr>
<tr>
<td>4</td>
<td>3.23</td>
<td>2.15</td>
<td>2.51</td>
<td>2.51</td>
<td>3.41</td>
<td>1.97</td>
<td>3.05</td>
<td>2.69</td>
</tr>
<tr>
<td>5</td>
<td>1.43</td>
<td>1.79</td>
<td>1.43</td>
<td>1.25</td>
<td>1.43</td>
<td>1.43</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>6-10</td>
<td>3.76</td>
<td>3.76</td>
<td>3.94</td>
<td>3.58</td>
<td>5.02</td>
<td>4.48</td>
<td>4.12</td>
<td>3.05</td>
</tr>
</tbody>
</table>

Panel B: Cumulative fractions of bankruptcies captured within deciles ranked by model forecasts (%)

<table>
<thead>
<tr>
<th>Decile</th>
<th>S01 Model</th>
<th>Augmented S01 Model</th>
<th>CJ04 Model</th>
<th>Augmented CJ04 Model</th>
<th>DSW07-S Model</th>
<th>Augmented DSW07-S Model</th>
<th>BS08 Model</th>
<th>Augmented BS08 Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74.01</td>
<td>74.91</td>
<td>75.63</td>
<td>76.34</td>
<td>71.15</td>
<td>72.22</td>
<td>74.73</td>
<td>75.45</td>
</tr>
<tr>
<td>2</td>
<td>85.48</td>
<td>86.74</td>
<td>86.92</td>
<td>87.63</td>
<td>85.48</td>
<td>87.10</td>
<td>87.46</td>
<td>88.71</td>
</tr>
<tr>
<td>3</td>
<td>91.58</td>
<td>92.29</td>
<td>92.11</td>
<td>92.65</td>
<td>90.14</td>
<td>92.11</td>
<td>91.58</td>
<td>93.01</td>
</tr>
<tr>
<td>4</td>
<td>94.80</td>
<td>94.44</td>
<td>94.62</td>
<td>95.16</td>
<td>93.55</td>
<td>94.09</td>
<td>94.62</td>
<td>95.70</td>
</tr>
<tr>
<td>5</td>
<td>96.24</td>
<td>96.24</td>
<td>96.06</td>
<td>96.42</td>
<td>94.98</td>
<td>95.52</td>
<td>95.88</td>
<td>96.95</td>
</tr>
<tr>
<td>6-10</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Robustness Checks

• Other proxies for the degree of noise (See the paper)

• “Private Firm” Empirical Setup:
  – Lower accounting report frequency: Compustat annual files (North America)
  – No market information
  – Potentially many outliers (small firms)
Robustness Check: “Private Firms” Setup

- **Improved** estimates
  - In Augmented Model, Coefficient on NI/TA is $[-0.05 - 0.09 \log(TA)]$
- **Improved** in-sample model fit
- **Substantially Improved** out-of-sample GINI: $0.57 \rightarrow 0.66$

### Full sample: 1979–2012, 290,811 firm-year observations, 2,537 bankruptcies

<table>
<thead>
<tr>
<th>Model</th>
<th>Variable</th>
<th>Full Sample Estimate</th>
<th>$-2 \log L$</th>
<th>Average of Out-of-Sample AUC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Firm Model</td>
<td>NI/TA</td>
<td>-0.02</td>
<td>45,983</td>
<td>0.7882</td>
</tr>
<tr>
<td></td>
<td>TL/TA</td>
<td>0.24***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Augmented Private Firm Model</td>
<td>NI/TA</td>
<td>-0.05**</td>
<td>44,596</td>
<td>0.8304</td>
</tr>
<tr>
<td></td>
<td>TL/TA</td>
<td>0.17***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>log(TA)</td>
<td>-0.12***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NI/TA*log(TA)</td>
<td>-0.09***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TL/TA*log(TA)</td>
<td>0.14***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference (= Private Firm Model - Augmented Model)</td>
<td></td>
<td>1,337</td>
<td>0.0422</td>
<td></td>
</tr>
<tr>
<td>$p$-value from a $\chi^2$-test</td>
<td></td>
<td>$&lt; 0.0001$</td>
<td>$&lt; 0.0001$</td>
<td></td>
</tr>
</tbody>
</table>