Multi-CoVaR and Shapley value: A Systemic Risk Measure

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First version: October 08, 2010, This version: October 04, 2014

Abstract

In this paper I propose a systemic risk measure to efficiently capture the systemic importance of each financial institution within a given system. The term systemic risk refers to the contagion risk to which each bank contributes to the financial system. The whole procedure is split into two parts: 1) calculate the total systemic risk, 2) use an allocation rule to attribute the total risk to each financial institution. For the first step, I define a measure “Multi-CoVaR” to estimate the total systemic risk, which also measures the institution’s marginal contribution to the systemic risk given that a set of institutions is in distress. For the second step, I apply the Shapley value methodology to allocate the total systemic risk. The additivity property of Shapley value ensures that the macroprudential tool can be efficiently implemented at the individual level.

Keywords: Systemic Importance, Contagion Risk, Shapley Value, Macroprudential Regulation

*This work is done when I was at Banque de France DSF-SMF. Special thanks go to Jean-Charles Rochet and Tobias Adrian for encouraging me to complete this work. The author also thank Jon Danielsson for helpful discussion at Pierre Marie Curie financial risk conference in Konstanz 2013. E-mail: zhili.cao@axa-im.com.
1 Introduction

The recent financial crisis has demonstrated the adverse effects of a large scale breakdown of financial intermediation both for other banks as well as for the rest of the economy. This is embodied in after the failure of Lehman Brother, the entire financial market has become extremely volatile, at the same time, the manufacture industry was deeply impacted. The concern that a specific bank defaults would trigger a domino effect in the financial sector is often brought forward to justify large scale government intervention and bailouts of failed institutions. But this increases the burden on taxpayers and sows the seeds of the next crisis.\footnote{See Farhi and Tirole (2012).} The failure of a bank causes these externalities because financial institutions are directly linked through interbank loans and derivatives. However, before the 2007-2009 financial crisis, banking regulation was based on individual risk, that is to say the adverse consequences that a bank brings for other banks as well as the economy as a whole was not considered by the regulator.

Moving towards a new regulation framework, economic researchers and regulators propose to implement a \emph{macroprudential} policy, which aims to mitigate the risk of the financial system as a whole (systemic risk) and to stabilize the financial system. To efficiently regulate systemic risk in banking sector need two steps: 1) measure each institution’s marginal systemic risk contribution to the whole system; 2) based on these measures, propose an efficient requirement standard to resist the systemic risk.\footnote{Efficient requirement standard refers to the total systemic requirement can be implemented at individual level without over or under-regulation.} In this paper, I mainly focus on the first step of macro-prudential regulation and propose a new methodology to measure the institution’s marginal systemic risk contribution.

The methodology proposed in this paper is complementary to Adrian and Brunnermeier (2011) CoVaR methodology, which is one of the most popular systemic risk measures in the literature. In Adrian and Brunnermeier (2011), authors use $\Delta CoVaR$, which is the difference between the VaR of the financial system conditional on a given financial institution being in a tail event and the VaR of financial system conditional on this financial institution being in a normal state, to capture the marginal contribution of systemic risk. Note that the $VaR$ of the financial system conditional on a given financial institution being in a tail event is also known as CoVaR. However, a specific feature of modern financial crisis is that there may be several institutions in financial distress at the same time, as a result, it is difficult to accurately measure a
specific institution’s systemic risk contribution taken in isolation while the effect could channel through other financial institutions themselves in distress at the same time. As a consequence, I modify the standard $\Delta \text{CoVaR}$ to the multi-$\Delta \text{CoVaR}$. Unlike the standard $\Delta \text{CoVaR}$, multi-$\Delta \text{CoVaR}$ is the difference between the $\text{VaR}$ of financial system conditional on a given set of financial institutions being in a tail event and the $\text{VaR}$ of the financial system conditional on this set of financial institutions being in a normal state. With this complementary extension, multi-$\Delta \text{CoVaR}$ captures the systemic risk contribution of several distressed financial institutions at the same time; furthermore it can capture the marginal systemic risk contribution of an institution that just joined the “distressed club” by taking the difference of the multi-$\Delta \text{CoVaR}$ of the set including the new one to the club and the multi-$\Delta \text{CoVaR}$ of the original set. Henceforth, given the specific feature of modern financial crisis, multi-$\Delta \text{CoVaR}$ can provide very useful information to build a macroprudential regulation framework.

The property of multi-$\Delta \text{CoVaR}$ is very useful for regulators. First, it allows to calculate the total contribution of systemic risk in the financial system, with this total contribution, one can apply an allocating rule to attribute the total risk to each bank. This total systemic risk is measured by the systemic risk contribution when all the institutions in the system are in distress. This total systemic risk measure can be set as a benchmark for regulators, since the sum of each institution’s systemic risk can never surpass (less than) this benchmark, otherwise there is an over-regulation (under-regulation) in the industry which may hurt the real economy. Second, it allows to calculate the marginal contribution of a bank for a given set of institutions is already in distress. This has the advantage to inform the regulators that which distressed institution should be bailed-out more urgently during the financial crisis. For example, in September 2008, Lehman Brother and AIG are both in financial distress at the same time and then Federal reserve has decided to bailout AIG. However, based on multi-$\text{CoVaR}$ measure one can calculate the marginal contribution for both LB and AIG and to bailout the one which has a larger marginal contribution of systemic risk. The third, the multi-$\text{CoVaR}$ can provide the systemic risk contribution of different groups. Again, this property is very useful for the regulators. Put the set of institutions differently for various sectors such as deposit banking sector, insurance sector, investment banking sector or government-sponsored enterprise (GSE), one can obtain the systemic risk contribution for these different sectors and regulate each sector appropriately.

Another advantage of multi-$\Delta \text{CoVaR}$ is that one can apply to Shapley value
methodology, which satisfies a set of axioms, to allocate systemic risk to each financial institution. The Shapley value methodology was initially proposed in the circumstance of cooperative games by Shapley (1953), in which a group of players generates a shared “value” (e.g. wealth, cost) for a group as a whole. The Shapley value of a player in a game turns out to be his expected marginal contribution over the set of all permutations on the set of players. This methodology can be applied in my setting, where financial institutions are connected via correlated high risk activities that trigger systemic risk in the system. The “value” mentioned above is the systemic risk generated by financial institutions. The additivity axiom of Shapley value states that the sum of each institution’s Shapley value of systemic risk contribution is exactly equal to the multi-$\Delta CoVaR$ of all the financial institutions in the system being in financial distress, which means that the Shapley value methodology allocates overall systemic risk in an efficient way. This coincides with Gourieroux and Monfort (2011), who suggest that systemic risk measures should be additive. While in the contrary, the sum of each standard $\Delta CoVaR$ is larger then the multi-$\Delta CoVaR$ of all the financial institutions in the system being in financial distress; that is to say, if I regulate financial institutions with the standard $\Delta CoVaR$, I implicitly punish the whole financial system, since the regulation is based on some systemic risk which does not even exist. Therefore, it would decrease the credit volume of banks and hurt the real economy.

After introducing the allocation methodology of how to distribute the systemic risk to each financial institution, I now turn to the estimation of the multi-$\Delta CoVaR$. There are many possible ways to calculate it. In this paper, I directly use the market returns of the financial system and of individual financial institutions for multi-$\Delta CoVaR$ calculation. For simplicity, at the very beginning, I assume the returns of the financial system and financial institutions follow a multi-t distribution to construct the distribution of financial system’s return conditional on the returns of the financial institutions. To that purpose, I use GARCH model and Dynamic Conditional correlation (DCC) (Engle (2002)) to construct the variance-covariance matrix of the conditional distribution. I loosen this assumption to calculate individual financial institution’s $VaR$: instead I use a nonparametric bootstrap technology implemented in GARCH model proposed by Pascual, Nieto, and Ruiz (2006) and Christofferson

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3. The normal distribution has been criticized since its quantile function doesn’t capture the fat tail events. I try to make distribution assumption as less as possible. In this case, I have to assume the returns follow a multi-t distribution, however, I loosen this assumption when I calculate the individual VaR.
and GonCalves (2005). With all the ingredient I need, then I apply Shapley value methodology to allocate systemic to each financial institution.

I apply the methodology of calculating the systemic importance in two banking panels, French panel and Chinese panel. Given these two banking systems, my measure provides the total systemic risk in the financial system and marginal contribution of systemic risk for each financial institution. I find that for both French and Chinese panels the overall systemic risk during the 2007-2009 financial crisis is larger than total systemic risk during the European sovereign debt crisis at the peak, and Chinese banks have little impact from the European crisis.

The remainder of the paper is organized as follows. I provide a literature review in section 2. In section 3, I develop the methodology, illustrate the differences with existing measures, show the properties of my measure and provide the calculation method of the measure. In section 4, I present the econometric framework and estimation. Section 5 gives the results and show the importance of additivity. In section 6, I briefly propose a regulation tool based on this measure. Section 7 concludes.

2 Literature review

This paper belongs to a growing empirical literature on systemic risk. The main contribution of the paper is that I focus on the aggregate contagion risk of a set of financial institutions and use an efficient allocation rule to attribute this aggregate contagion risk to each financial institution in order to determine their systemic importance. The efficient allocation means that the overall aggregate contagion risk is all attributed to each financial institution without waste and won’t be attributed by any risks that don’t exist, hence we have that the sum of attributed risk to each financial institution equals the overall aggregate contagion risk.

The most of empirical literature have particular interest are those that treat explicitly the financial system as a portfolio of institutions. Lehar (2005) uses a sample of international banks from 1988 to 2002, to estimate the dynamics and correlations between bank asset portfolios. After the 2007-2009 financial crisis, two strands of the literature have been emerged. The first strand of the literature captures the bank's sensitivity to a systematic shock in the whole financial system. In Huang, Zhou,

4. Some papers are focusing on the exposure of banks when a systematic shock happens, e.g. Huang, Zhou, and Zhu (2010); Brownlees and Engle (2010); others are focusing on the contagion, e.g. Segoviano and Goodhart (2009); Billion, Getmansky, Lo, and Pelizzon (2010); however Greenwood, Landier, and Thesmard (2011) allows to distinguish these two risks.
and Zhu (2010), they compute first the total market loss when a systematic shock takes place and then attribute this loss to each financial institution to capture the systemic importance. Similarly, Acharya, Pederson, Phillipon, and Richardson (2010) proposed a measure called “Marginal Expected Shortfall” (MES); this measure allows to investigate what is the loss of one specific institution in case of a large loss in the financial system. From a theoretical point of view, it is akin to a Beta computed within the Capital Asset Pricing Model (CAPM). The difference rests on the fact that the MES measures a so-called “tail-Beta”, which is a projection of institution’s return to a space where the market return is lower than a specific threshold. In Brownlees and Engle (2010), the authors construct an econometric framework to compute the MES. In their model, the MES is consist of 3 parts: the volatility of institutions’ market capitalization return, their correlation and the conditional tail expectation. Adrian and Brunnermeier (2011) also suggest a measure exposure-\(CoVaR\) to capture this effect. Greenwood, Landier, and Thesmard (2011) propose a measure to gauge a bank’s exposure to sector-wide deleveraging. This specific measure is focusing on the fire-sale effect in the financial market.

Another strand of the literature gauges systemic risk by the contagion through the financial market. This contagion is twofold. On the one hand, this contagion could be considered as the bilateral contagion, which means one bank’s sensitivity to a shock of another bank and vice versa. Segoviano and Goodhart (2009) propose a methodology to capture such inter-linkages contagion. The objective of the methodology is to find the joint distribution, which best fits a prior joint distribution according to the information criteria and is consistent with the probability of distress for each bank. This gives the probability of default of one bank given another bank is in distress. In Billion, Getmansky, Lo, and Pelizzon (2010), they propose a measure of connectedness based on principal-component analysis and Granger-causality networks. They find that financial institutions are highly interrelated over the past decade. On the other hand, this contagion could be considered as the contagion of a bank to the whole financial system, however, one can argue that this is also the contribution of financial institutions to overall systemic risk. Adrian and Brunnermeier (2011) use the \(CoVaR\) measure to characterize one bank’s systemic risk contribution to the financial system. In their framework, this measure is the Value at Risk of the whole financial sector conditional on a given institution being in distress. Tarashev, Borio, and Tsatsaronis (2010) and Drehmann and Tarashev (2011) use individual risk measures to capture an institution’s systemic importance by implementing Shapley value methodology; they
also focus on the contribution of institutions to systemic risk. However, in this paper, I use a systemic risk measure to capture the systemic importance of institutions and then allocate the measure by Shapley value methodology. In Greenwood, Landier, and Thesmard (2011), their method also allows to capture one bank’s contribution to the overall deleveraging risk, but, again, their measure is specifically focused on the fire-sale effect.

There are also two very useful survey about the systemic risk measures Benoit, Colletaz, Hurlin, and Perignon (2012) and Bisias, Flood, Lo, and Valavanis (2012).

3 Multi-CoVaR and Shapley value methodology

In this section, I present how the Shapley value methodology can be applied to the CoVaR measure and its properties.

3.1 Remind CoVaR

Recall that Value-at-Risk (VaR) is defined as the solution to

$$\mathbb{P}(r_t \leq VaR^q_t) = q,$$

$VaR^q_t$ is the $q$-quantile of the return $r_t$. Note that, with this definition, $VaR^q_t$ is typically a negative number.

Adrian and Brunnermeier (2011) defined $CoVaR_{q, t}^{sys|i}$ as the $VaR$ of the financial system conditional on some event $C(r^i_t)$ of institution $i$ at time $t$. That is, the $CoVaR_{q, t}^{sys|i}$ for financial system and confidence level $q$ when the institution $i$ is on some event $C(r^i_t)$ at time $t$ is defined by:

$$\mathbb{P}(r_t^{sys} \leq CoVaR_{q, t}^{sys|C(r^i_t)}|C(r^i_t)) = q,$$ (1)

and institution $i$’s contribution to system is measured by:

$$\Delta CoVaR_{q, t}^{sys|i} = CoVaR_{q, t}^{sys|r^i_t \in \{\text{adverse case}^i\}} - CoVaR_{q, t}^{sys|r^i_t \in \{\text{normal case}^i\}}.$$ (2)

In practice, they focus on $\{r^i_t = VaR^i_t\}$ as the conditioning event and simplify the notation $CoVaR_{q}^{sys|r^i_t = VaR^i_t} = CoVaR^i_q$ and $\Delta CoVaR_{q}^{sys|i} = \Delta CoVaR^i_q$, meanwhile they focus on $\{r^i_t = \text{Median}^i\}$ in the normal case as the conditioning event, $CoVaR_{q, t}^{sys|r^i_t = \text{Median}^i}$. Hence, $\Delta CoVaR^i_q$ denotes the difference between the $VaR$ of
financial system conditional on the financial institution \(i\) being in a tail event and the \(VaR\) of financial system conditional on the financial institution \(i\) being in a normal state. Note that, they also define the system returns as the weighted sum of individual returns at each time \(t\).

3.2 Multi-CoVaR

After reminding the definition of CoVaR of Adrian and Brunnermeier (2011) above. They propose \(\Delta CoVaR\) as a systemic measure to quantify the risk spillover effects, what is the impact on the financial system if one specific institution is in financial distress. However, during the financial crisis, I have seen that several financial institutions may have been in financial distress at the same time, therefore an interesting extension of CoVaR can be defined as follow:

**Definition 1.** I denote by \(CoVaR_{1,\ldots,S}^{q,t}\) the \(VaR\) of the financial system conditional on some event \(\{C(r_1^t), \ldots, C(r_S^t)\}\) of a set of institutions \(\{1, \ldots, S\}\) at time \(t\). That is \(CoVaR_{1,\ldots,S}^{q,t}\) for financial system and confidence level \(q\) when the set of institutions \(\{1, \ldots, S\}\) is on some event \(\{C(r_1^t), \ldots, C(r_S^t)\}\) at time \(t\) is defined by:

\[
P(r_{sys}^t \leq CoVaR_{q,t}^{1,\ldots,S} | C(r_1^t), \ldots, C(r_S^t)) = q, \tag{3}
\]

and the set of institutions \(\{1, 2, \ldots, S\}\)’s contribution to financial system is denoted by:

\[
\Delta CoVaR_{1,\ldots,S}^{q,t} = CoVaR_{q,t}^{1,\ldots,S}_{r_i^t \leq VaR_{r_i^t}, \ldots, r_S^t \leq VaR_{r_S^t}} - CoVaR_{q,t}^{1,\ldots,S}_{-\alpha \sigma_i^t \leq r_i^t \leq \alpha \sigma_i^t, \ldots, -\alpha \sigma_S^t \leq r_S^t \leq \alpha \sigma_S^t}. \tag{4}
\]

Unlike above, I focus on \(\{r_i^t \leq VaR_{r_i^t}\}\), for \(i = 1, \ldots, S\), in the adverse case as the conditioning event and simplify the notation, \(CoVaR_{q,t}^{sys|r_i^t \leq VaR_{r_i^t}, \ldots, r_S^t \leq VaR_{r_S^t}} = ACoVaR_{q,t}^{S}\); I focus on \(\{-\alpha \sigma_i^t \leq r_i^t \leq \alpha \sigma_i^t\}\), for \(i = 1, \ldots, S\), in the normal case as the conditioning event and simplify the notation, \(CoVaR_{q,t}^{sys|\phantom{r_i^t}|\ldots|\phantom{r_i^t}|\phantom{r_i^t}|\ldots|\phantom{r_i^t}} = NCoVaR_{q,t}^{S}\). Note that the normal case is characterized as an \(\alpha\)-standard deviation around the mode, where I assume the mode is 0 and \(\sigma_i^t\) is the conditional standard deviation of institution \(i\). It is straightforward to see that, \(\Delta CoVaR_{q,t}^{sys|1,\ldots,S} = ACoVaR_{q,t}^{S} - NCoVaR_{q,t}^{S}\), letting \(\Delta CoVaR_{q,t}^{sys|1,\ldots,S} = \Delta CoVaR_{q,t}^{S}\). Hence \(\Delta CoVaR_{q,t}^{S}\) denotes the difference between the \(VaR\) of financial system conditional on a set of financial institutions \(\{1, \ldots, S\}\) being in a tail event and the \(VaR\) of financial system conditional on the set of financial institutions \(\{1, \ldots, S\}\) being in a normal state at
time $t$.  

The economics of multi-$CoVaR$ are quite similar to those of standard $CoVaR$. Both of them quantifies the spillover effects by measuring institution(s) add(s) to the global risk of the financial system. The spillover effects are embodied in several ways. First, if several banks are selling off their mark-to-market assets to meet their obligations, it will lower the price of these assets and further decrease the values of the banks who hold these assets, consequently it will hurt market liquidity and harm banks ability of raising new fund and even more likely trigger the insolvency problem of banks. This implies the second spillover effect, when several banks meet the insolvency problem at the same time, it will drag other banks who hold direct debt contract of distressed banks into trouble, and this is so called domino effect. Last, when GSIFIs are insolvent, the government will intervene to bail-out these banks due to the “too-big-to-fail” effect and increase the burden of taxpayers and sow the seeds for the next crisis. Note that, this multi-$CoVaR$ captures spillover effects, and measure the contribution of systemic risk of a set of banks when it is in distress.

3.2.1 Calculation of Multi-$CoVaR$

As to the $VaR$ for individual institutions, I compute $ACoVaR_{q,t}^S$ and $NCoVaR_{q,t}^S$ for a set of institutions in the same spirit in order construct the $\Delta CoVaR_{q,t}^S$ measure. The equation (3) can be reformulate as:

$$
\frac{P(r_{t}^{s_{gs}} \leq CoVaR_{q,t}^{R_{1}^{1},\ldots,R_{q,t}^{S}}, C(r_{t}^{1}), \ldots, C(r_{t}^{S}))}{P(C(r_{t}^{1}), \ldots, C(r_{t}^{S}))} = q. \tag{5}
$$

To compute $ACoVaR_{q,t}^S$, I replace the conditional event $C(r_{t}^{i})$ by $\{r_{t}^{i} \leq VaR_{q,t}^{i}\}$. Since the individual risk measure $VaR_{q,t}^{i}$ can be easily obtained, I can calculate the denominator of equation (5), which generates a joint probability $q_{d}$, (d for denominator),

$$
\int_{-\infty}^{VaR_{q,t}^{1}} \cdots \int_{-\infty}^{VaR_{q,t}^{S}} D_{S,t}(r_{1}^{1}, \ldots, r_{t}^{S}) dr_{1}^{1} \cdots dr_{t}^{S} = q_{d}, \tag{6}
$$

where $D_{S,t}(\cdot)$ denotes the probability density function of a $S$-dimensional random

5. Note that this is different from the initial definition of Adrian and Brunnermeier (2011), where they let the return exactly equals to its $VaR$ as the adverse case and the return exactly equals to its median as the normal case. See also Ergun and Girardi (2012), they have changed the initial definition of conditional event in the standard $CoVaR$.

6. I will present how to calculate the individual $VaR$ in the next section.
vector \( r_{S,t} = (r_{1,t}^1, \ldots, r_{t}^S)' \). Hence, the numerator in the equation (5) can be rewritten in the same way,

\[
\int_{-\infty}^{ACoVaR_{q,t}^S} \int_{-\infty}^{VaR_{1,t}^1} \cdots \int_{-\infty}^{VaR_{S,t}^S} D_{S+1,t}(r_{sys}^1, r_{1,t}^1, \ldots, r_{t}^S)dr_{sys}^1dr_{1,t}^1 \cdots dr_{t}^S = q \times qd, \tag{7}
\]

where \( D_{S+1,t}(\cdot) \) denotes the probability density function of a \( S+1 \)-dimensional random vector \( r_{S+1,t} = (r_{sys}^1, r_{1,t}^1, \ldots, r_{t}^S)' \). As the \( ACoVaR_{q,t}^S \) is the only unknown in the equation (7), it can be solved numerically.

Regarding to the \( NCoVaR_{q,t}^S \), the conditional event is \( \{-\alpha \sigma_i^t \leq r_i^t \leq \alpha \sigma_i^t\} \) and the conditional standard deviation \( \sigma_i^t \) can be obtained by implementing a GARCH process. The denominator of equation (5), in this case, gives,

\[
\int_{-\infty}^{\alpha \sigma_i^1} \cdots \int_{-\infty}^{\alpha \sigma_i^S} D_{S,t}(r_{1,t}^1, \ldots, r_{t}^S)dr_{1,t}^1 \cdots dr_{t}^S = p_d, \tag{8}
\]

and the numerator gives,

\[
\int_{-\infty}^{NCoVaR_{q,t}^S} \int_{-\infty}^{\alpha \sigma_i^1} \cdots \int_{-\infty}^{\alpha \sigma_i^S} D_{S+1,t}(r_{1,t}^1, \ldots, r_{t}^S)dr_{1,t}^1 \cdots dr_{t}^S = q \times p_d, \tag{9}
\]

therefore, I can compute \( NCoVaR_{q,t}^S \) with the same procedure as for \( ACoVaR_{q,t}^S \), thus the \( \Delta CoVaR_{q,t}^S \) is obtained by taking the difference between \( ACoVaR_{q,t}^S \) and \( NCoVaR_{q,t}^S \).

### 3.2.2 Properties of Multi-CoVaR

This measure has three advantages. First, it allows to calculate the total contribution of systemic risk in the financial system, with this total contribution, one can apply an allocating rule to attribute the total risk to each bank. This total systemic risk is measured by the systemic risk contribution when all the institutions in the system are in distress. Suppose there are \( N \) banks in the system, the total contribution of systemic risk is given by:

\[
\Pr(R_{sys} \leq CoVaR_{q,t}^{1,\ldots,N}|C(r_{1}^1), \ldots, C(r_{N}^N)) = q, \tag{10}
\]

and

\[
\Delta CoVaR_{q,t}^{N} = ACoVaR_{q,t}^{N} - NCoVaR_{q,t}^{N}. \tag{11}
\]
Note that the equation (10) characterizes an extreme case in the system, where all the institutions are in financial distress, and the equation (11) gives the related systemic risk contribution in this scenario, which is the overall systemic risk contribution in the system. This total systemic risk measure can be set as a benchmark for regulators, since the sum of each institution’s systemic risk can never surpass (less than) this benchmark, otherwise there is an over-regulation (under-regulation) in the industry which may hurt the real economy.

Secondly, it allows to calculate the marginal contribution of bank $i$ for a given set of institutions $S$ is already in distress. Denote $\Delta_i(S)$ the marginal systemic risk contribution of one specific institution $i$ is determined by:

$$
\Delta_i(S) = \Delta CoVaR(S \cup \{i\}) - \Delta CoVaR(S), \text{ for } S \subset N, \ i \notin S.
$$

The equation (12) has the advantage to inform the regulators that which distressed institution should be bailed-out more urgently during the financial crisis. For example, in September 2008, Lehman Brother and AIG are both in financial distress at the same time and then Federal reserve has decided to bailout AIG. However, based on (12) one can calculate $\Delta(S)$ for both LB and AIG and to bailout the one which has a larger marginal contribution of systemic risk.

The last, the multi-$CoVaR$ can provide the systemic risk contribution of different groups. Again, this property is very useful for the regulators. Put the set of institutions $S$ differently for various sectors in the financial system as deposit banking sector, insurance sector, investment banking sector or government-sponsored enterprise (GSE), one can obtain the systemic risk contribution for these different sectors and regulate each sector appropriately. Moreover, we can put the different set of institutions as different regions or different countries to analyze a geographical distinction of systemic risk contribution to better inform the international establishment as OECD, IMF, and BIS etc.

On top of that, this extension about standard $CoVaR$ allows us to implement Shapley value methodology to attribute the systemic risk to each financial institution in the system, which has some related advantages to the design of prudential regulation issue.


3.3 Shapley value

In this paper, the Shapley value plays a role as a systemic risk distributor, which means that I use Shapley value methodology as an allocation rule to assign the overall systemic risk contribution, as in (11), to each institution in the financial system. An introduction of Shapley value is presented below.

The Shapley value methodology was initially proposed in the circumstance of cooperative games, in which a group of players generates a share “value” (e.g. wealth, cost) for a group as a whole. The Shapley value of a player in a game turns out to be his expected marginal contribution over the set of all permutations on the set of players. For example, a group of agents would like to connect to a server in order to benefit a high speed functioning of their own PC, however, the maintain of the server is costly, the Shapley value is a fair and efficient allocation rule to share the maintaining costs among agents and the Shapley value of an agent is his expected marginal contribution over all possible set of the agents. This methodology can be applied in our case, where financial institutions are connected in financial market with high correlated risk activities that trigger systemic risk in the system. The “value” mentioned above is the systemic risk generated by financial institutions (banks).

In order to apply the Shapley value methodology to a financial system, it is sufficient to define a so-called “characteristic function”. As mentioned above, I define this “characteristic function” as Multi-CoVaR. This function is the same over the set of all permutations on the set of banks and map each subsystem into a risk measure. The characteristic function, \( v \), should accept as input anyone of the \( 2^N - 1 \) subsystems of banks and should deliver the system-wide risk measures when applied to the entire system.

The derivation of Shapley values involves the following process.

There are \( N \) players in a superadditive game, which are financial institutions. Let \( v : 2^N \to R^+ \) be a function defining the systemic risk for each subset of \( N \), and \( v(\phi) = 0 \). The objective is to find non-negative systemic risk attribution \( \{Sh_i\}_{i\in N} \) such that:

- Axiom 1 (Additivity/Efficiency): \( \sum_{i\in N} Sh_i = v(N) \),
- Axiom 2 (Dummy axiom): If \( i \) is such that \( \Delta_i(S) = v(\{i\}) \) for all \( S \) such that \( i \notin S \), then \( Sh_i = v(\{i\}) \),
- Axiom 3 (Symmetry): If \( i \neq j \) such that \( \Delta_i(S) = \Delta_j(S) \) for all \( S \) such that \( i, j \notin S \), then \( Sh_i = Sh_j \).

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7. Since the empty set \( \phi \) does not play a role in contributing the value.
Axiom 4 (Linearity): Suppose $v(S) = v_1(S) + v_2(S)$ where $v_1$ and $v_2$ and assume $v_1(\phi) = v_2(\phi) = 0$, and $\{Sh^1_i\}_i$ are systemic risk shares for $v_1$-risk and $\{Sh^2_i\}_i$ are systemic risk shares for $v_2$-risk, then $Sh_i = Sh^1_i + Sh^2_i$, for all $i$, defines the systemic risk shares for $v$-risk.

There is a unique way to satisfy axioms 1, 2, 3 and 4, called Shapley value and Shapley value for bank $i$ is:

$$Sh_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!}(v(S \cup \{i\}) - v(S)),$$

(13)

where $n$ is the total number of banks and the sum extends over all subsets $S$ of $N$ not containing bank $i$. This formula can be interpreted as the expected marginal contribution of bank $i$ over the set of all permutations on the set of banks.

Now, I am going to talk about the economic meaning of these axioms. Axiom 1 states that the sum of of Shapley value equals the aggregated systemic risk, this additivity property also called efficiency property, since the total systemic risk can be attributed among banks with no loss and with no gain. This property has a big advantage in macro-prudential policy, if macro-prudential tools are based on some sub(super)-additivity systemic risk measures, given the linear relationship between the macro-prudential regulation and systemic regulation, supervisors will punish (subsidize) the economy for no reason. Axiom 2 says that if the systemic risk of $i$ is orthogonal of any other $j \neq i$'s systemic risks, then the systemic risk share of $i$ should be exactly equals to its systemic risk alone. This is the case where standard CoVaR coincide with Shapley value methodology, whereas the systemic risk of $i$ is not orthogonal of any other institutions’ systemic risks, this is the reason why the standard CoVaR is different from Shapley value methodology. Axiom 3 enforces the fairness among players, for any two different banks, if their marginal systemic risk contribution is the same for any subsets $S \subset N$, then their Shapley value should be the same, therefore, they should be charged the same as well. The last axiom suggests that this systemic risk can be decomposed into two independent risks, for example these two risks are from two different services in the bank. Then to obtain the systemic risk shares it is sufficient to calculate the risk of each services and take the sum the the two. If one can decompose this systemic risk, I can apply the Shapley value methodology in a decentralized way.
4 Estimation

There are several ways to calculate CoVaR. Adrian and Brunnermeier (2011) propose to use quantile regression to estimate time-varying ΔCoVaR and VaR. They use a set of state variables as regressors to estimate q-quantile parameters to fit estimated CoVaR and VaR, a big advantage of this method is that they do not consider any specific distribution on random variables in order to obtain the time-varying CoVaRt and VaRt. In this paper, I use another econometric framework to illustrate the methodology presented above. In order to avoid the process of which state variables should be selected, I work directly with the returns of the individuals. To clarify the idea, I firstly identify which ingredients are indispensable for calculating multi-CoVaR: 1) a probability density function of the returns (both system and individuals), 2) the conditional volatility of individual returns to define the so called “normal case”, 3) the idiosyncratic risk VaR to characterize the “adverse case” for individuals.

4.1 Multi-t distribution

From the equations (6)-(9), the key ingredient in computing multi-CoVaR is the joint probability density function. An important stylized fact in financial market is that the asset returns often have fatter tails than normal distribution, and I assume the vector \( r_{S+1,t} \) follows a multi-t distribution with mean 0 and variance-covariance \( \Sigma_t \) (\( v_t \) for degree of freedom):

\[
\begin{pmatrix}
    r^{sys}_t \\
r^1_t \\
\vdots \\
r^S_t
\end{pmatrix}
\sim t_{v_t}

\begin{pmatrix}
    0 & & & & \\
    0 & \sigma^2_{sys,t} & \rho_{sys1,t}\sigma_{sys,t}\sigma_{1,t} & \cdots & \rho_{sysS,t}\sigma_{sys,t}\sigma_{S,t} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & \rho_{SYS,t}\sigma_{SYS,t}\sigma_{S,t} & \rho_{S1,t}\sigma_{S,t}\sigma_{1,t} & \cdots & \sigma^2_{S,t}
\end{pmatrix},
\]

where \( \sigma_{i,t} \) denotes the conditional volatility of \( i \) and \( \rho_{ij,t} \) denotes the conditional correlation between \( i \) and \( j \). Therefore, to estimate the multi-t distribution in each time \( t \), it is sufficient to estimate the conditional volatilities and conditional correlations. Note that, this is the only case where I make a distribution assumption to calculate the multi-CoVaR.

---

8. For different state variables selected, the results of quantile estimation may be different.
4.2 Volatility

As mentioned before, I need to compute individual’s conditional volatility $\sigma_{i,t}$ at first place to calculate $NCoVaR_{q,t}$. To do so, I use the GARCH process. In wide world of GARCH specifications, TGARCH is picked to model volatility to capture so called “leverage effect”, which has the fact that a negative return increases variance by more than a positive return of the same magnitude. I also a diagnosis of this GARCH model to show that the model is well specified in the Appendix. The evolution of the conditional variance dynamics in this model is given by:

\[
\begin{align*}
    r_t &= \epsilon_t \sigma_t, \\
    \sigma_t^2 &= \omega_G + \alpha_G r_{t-1}^2 + \gamma_G r_{t-1}^{-1} I_{t-1}^- + \beta_G \sigma_{t-1}^2,
\end{align*}
\]

with $I_{t-1}^- = r_t \leq 0$. The model is estimated by Quasi-MLE which guarantees the consistency of the estimator. I have:

\[
\sqrt{T}(\hat{\theta}_{MLE} - \theta^o) \sim N(0, J_0^{-1} I_0^{-1} J_0^{-1}),
\]

with $\hat{I} = \frac{1}{T} \sum S_t(\hat{\theta}) S'_t(\hat{\theta})$ and $\hat{J} = \frac{1}{T} \sum \frac{\partial^2 \log L}{\partial \theta \partial \theta'}$ is the Hessian of total log-likelihood function. Where $\theta = (\alpha_G, \gamma_G, \beta_G)$.

4.3 Correlations

The time varying correlation is modeled by using DCC approach. Correlation matrix is given by:

\[
P_t = diag(Q_t)^{-\frac{1}{2}} Q_t diag(Q_t)^{-\frac{1}{2}},
\]

where $Q$ is a so called pseudo-correlation matrix and it is positive defined. The DCC specification is defined as:

\[
Q_t = (1 - \alpha_C - \beta_C) \overline{Q} + \alpha_C \epsilon_{t-1}^* \epsilon_{t-1}^* + \beta_C Q_{t-1},
\]

where $\epsilon_t^*$ is the standardized returns with $\epsilon_t^* = diag(Q) \times \epsilon_t$, $\overline{Q}$ is an intercept matrix with $\overline{Q} = \mathbb{E}[\epsilon_t^* \epsilon_t^*]$. I use the estimation method mentioned above for this dynamic conditional correlation.
4.4 Value-at-Risk

Instead of using a distribution based approach to calculate VaR, here, I loosen the multi-t distribution assumption and use non-parametric bootstrap methodology to determine individuals VaR. The nonparametric bootstrap allows us to estimate the sampling distribution of a statistic empirically without making assumptions about the form of population, and without deriving the sampling distribution explicitly. The key bootstrap concept is that the population is to the sample as the sample is to the bootstrap sample. Then, I proceed the bootstrap technique in the following way.

For a given institution series of returns \( \{ r_{i,1}, ..., r_{i,T} \} \), consider a TGARCH model as in the previous case, whose parameters have been estimated by Quasi-MLE. Then I can obtain the standardized residuals, \( \hat{\epsilon}_{i,t} = \frac{r_{i,t}}{\hat{\sigma}_{i,t}} \), where \( \hat{\sigma}_{i,t}^2 = \hat{\omega}_G + \hat{\alpha}_G r_{i,t-1}^2 + \hat{\gamma}_G r_{i,t-1}^2 I_{i,t-1} + \hat{\beta}_G \hat{\sigma}_{i,t-1}^2 \), and \( \hat{\sigma}_{i,1}^2 \) is long-run variance of the sample.

To implement the bootstrap methodology, it is necessary to obtain bootstrap replicates \( R_{i,T}^* = \{ r_{i,1}^*, ..., r_{i,T}^* \} \) that mimic the structure of original series of size \( T \). \( R_{i,T}^* \) are obtained from following recursion (Pascual, Nieto, and Ruiz (2006)).

\[
\hat{\sigma}_{i,t}^* = \hat{\omega}_G^* + \hat{\alpha}_G^* r_{i,t-1}^* + \hat{\gamma}_G^* r_{i,t-1}^* I_{i,t-1}^* + \hat{\beta}_G^* \hat{\sigma}_{i,t-1}^* \]
\[
r_{i,t}^* = \hat{\epsilon}_{i,t}^* \hat{\sigma}_{i,t}^* \]

where \( \hat{\sigma}_{i,1}^* = \hat{\sigma}_{i,1}^2 \) and \( \hat{\epsilon}_{i,t}^* \) are random draws with replacement from the empirical distribution of standardized residuals \( \hat{\epsilon}_{i,t} \).\(^9\) This bootstrap method incorporate uncertainty in the dynamics of conditional variance in order to make useful to estimate VaR. Given the bootstrap series \( R_{i,T}^* \), I can obtain estimated bootstrap parameters, \( \{ \hat{\omega}_G^*, \hat{\alpha}_G^*, \hat{\gamma}_G^*, \hat{\beta}_G^* \} \). The bootstrap of historical values are obtained from following recursions

\[
\hat{\sigma}_{i,t}^{bs} = \hat{\omega}_G^{bs} + \hat{\alpha}_G^{bs} r_{i,t-1}^{bs} + \hat{\gamma}_G^{bs} r_{i,t-1}^{bs} I_{i,t-1}^{bs} + \hat{\beta}_G^{bs} \hat{\sigma}_{i,t-1}^{bs} \]
\[
r_{i,t}^{bs} = \hat{\epsilon}_{i,t}^{bs} \hat{\sigma}_{i,t}^{bs} \]

where \( \hat{\sigma}_1^{bs} \) is the long-run variance of the bootstrap sample \( R_{T}^{bs} \), note that the historical values is based the original series of return and on the bootstrap parameters. I repeat the above procedure \( B \) times, and estimated \( \hat{VaR}_{i}^{bs}(q) \) is \( k^{th} \)-order of series \( \hat{r}_{i}^{bs} \), for \( b = 1, ..., B \), where \( k = B \times q \).

\(^9\) It is necessary to sample with replacement, because one would otherwise simply reproduce the original sample.
5 Empirical Analysis

I apply the methodology and econometric framework described in the previous sections and examine the systemic risk in in France banking system and China banking system.

5.1 Data

I study two different banking panels in this paper, French panel and Chinese panel. I choose the top five French banks that are in the top 50 world banks which based on the market capitalization as of January 20, 2012, as the French banking system, from April 19, 2002 to June 29, 2012; and I also choose the top five Chinese banks that are in this top 50 world banks panel, as my Chinese banking system, from October 27, 2006 to June 29, 2012. I extract weekly returns and market capitalization from Bloomberg. The goal is to analyze the systemic risk in these two banking panels (table 1) in terms of market capitalization returns separately.\(^\text{10}\)

<table>
<thead>
<tr>
<th>French Panel</th>
<th>Ticker</th>
<th>Chinese Panel</th>
<th>Ticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP Paribas</td>
<td>BNP</td>
<td>Industrial &amp; Commercial Bank of China</td>
<td>ICBC</td>
</tr>
<tr>
<td>Société Générale</td>
<td>SoGen</td>
<td>China Construction Bank</td>
<td>CBC</td>
</tr>
<tr>
<td>Crédit Agricole</td>
<td>CrdAgl</td>
<td>Bank of China</td>
<td>BC</td>
</tr>
<tr>
<td>Natixis</td>
<td>Natixis</td>
<td>Bank of Communications</td>
<td>CCB</td>
</tr>
<tr>
<td>Crédit Mutuel</td>
<td>CrdMtl</td>
<td>China Merchants Bank</td>
<td>CMBC</td>
</tr>
</tbody>
</table>

Table 1: Banking Panel.

Figure 1 gives visual insights on the booms and busts of the French and Chinese banking system. The figure shows the cumulative system returns in both banking panels, from April 2002 to June 2012 for French panel (blue line) and from October 2006 to June 2012 for Chinese panel (red line). The blue line experiences a loss from the start to the mid 2003, and then had a steep growth between mid 2003 and June 2007. Starting from July 2007, the fall of financial market has been dramatic, with the large gains transforming into the huge losses. The system hit the bottom in March 2009 and start a slow recovery that is then interrupted by the 1st European crisis of May 2010 and 2nd European crisis during the summer 2011. The red line also had

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\(^\text{10}\) As There is no public data for BPCE (Group BPCE was founded in 2009) and Agriculture Bank of China (It was listed on the Hong Kong stock exchanges in July 2010) during the financial crisis 2007-2009, these two banks are not considered in this paper.
a steep growth between the beginning of the sample and the early 2008, however, they incurred the losses just several months after French panel faced the losses, this can be explained by the geographical transfer of crisis have some lags. The Chinese system hit the bottom in March 2009 and start a slow recovery that is interrupted only during the 2nd European debt crisis.

Figure 1: Cumulative system market capitalization returns of France panel and China panel.

Figure 2 and figure 3 show that QQ-plot of French panel and Chinese Panel. In line with the stylized fact of financial data, there are fat tails on the returns in both panels, therefore it is irrational to assume the returns of banks follow a multi-normal distribution, which justify my previous assumption that the returns follows a multi-t distribution. An interesting result found from these 2 figures is that French banks have much larger fat tails then Chinese banks, this may be explained by the French banks are more affected by the 2007-2009 financial crisis than Chinese banks do.

5.2 Full sample estimation result

I used the methodology introduced in Section 3 and section 4 to analyze the panels. TGARCH and DCC models are fitted on each bank over the whole sample period and in this section I report information on the parameter estimates, fitted series and check on the autocorrelations.

In table 2 I show all banks and the systems’ parameter estimates of the TGARCH
Figure 2: QQ-plot of market capitalization returns of French panel.

Figure 3: QQ-plot of market capitalization returns of Chinese panel.

(left side) and the DCC (right side) models for both panel, note that the DCC is each bank’s returns dynamic conditional correlation with the system returns. The TGARCH parameters do not fluctuate much, but for Chinese banks, they are less subject to the so called “leverage effect”. The point estimates are in line with the typical TGRACH estimates, with slightly higher $\alpha$s and $\gamma$s together with lower $\beta$s.
implying a higher level of unconditional kurtosis. Turning to the DCC, parameters are again close to the typical set of estimates. The diagnostic of autocorrelations is provided in Appendix.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_G$</th>
<th>$\gamma_G$</th>
<th>$\beta_G$</th>
<th>$\alpha_C$</th>
<th>$\beta_C$</th>
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<td></td>
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<tr>
<td>FRA. System</td>
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<table>
<thead>
<tr>
<th></th>
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<th>$\beta_G$</th>
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<th>$\beta_C$</th>
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</thead>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>0.05</td>
<td>0.85</td>
<td>0.00</td>
<td>0.98</td>
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<td>0.00</td>
<td>0.86</td>
<td>0.03</td>
<td>0.96</td>
</tr>
<tr>
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<td>0.24</td>
<td>0.74</td>
<td>0.00</td>
<td>0.98</td>
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<td>0.30</td>
<td>0.50</td>
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<td>CCB</td>
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<td>0.00</td>
<td>0.97</td>
<td>0.02</td>
<td>0.96</td>
</tr>
<tr>
<td>CMBC</td>
<td>0.06</td>
<td>0.02</td>
<td>0.92</td>
<td>0.02</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 2: TGARCH and DCC estimation results.

I provide the dynamics of volatility in figure 4. The blue line represents the time varying system volatility and the red line represents the average time varying of individuals volatilities of all banks in each panels. In Chinese panel, the system volatility is always lower than the average volatility, this is due to the diversification effect among banks. Moreover both average&system volatilities are relative high during the crisis period to the post crisis period, however, the magnitude of the volatilities are much lower than the French banks. Note that, these two panels behave very differently during the European crisis period, this is because the European debt crisis has a huge direct impact on French banks, but has little impact for Chinese banks. In French panel, during the calm period 2003-mid 2007, both system and average volatilities stay at a low level meanwhile there is a big spread between average volatility and system volatility; this is due to diversification effect when market is in a good period. However, this spread narrows down and volatility level increases sharply when financial crisis starts, this is because of the co-movement of individuals returns and systemic return when “things” are going bad. Therefore, it will be useful to investigate the correlations of returns among banks.

Figure 5 displays average correlation by banks in each panel. In French panel, there is an obvious trend of increasing in average correlation from calm period to
Figure 4: Average and system volatility of French system and Chinese system.

crisis period, which explains why the diversification effect plays little role during the financial crisis. This implies that the sub-additivity property of risk measures may not hold in some cases, in my view, especially for the systemic risk measures, because a large systemic risk often comes with a high correlation among the agent in the system. It’s the same for Chinese panel, the level of correlation increases after the crisis, but the magnitude of correlation in China is larger than in French.
Figure 5: Dynamic Conditional Correlation of French system and Chinese system.

Figure 6 provides the average individual Value-at-Risk in French panel and Chinese panel. The average individual VaRs are quite similar with the average volatility in term of evolution, which is in line with the model specification, the return is product of volatility and the innovation. In French panel, during the calm period, the level

11. Throughout the whole paper, the confidence level is set as 5%.
of individual VaR is low, because the volatility is low. It sharply increased after the crisis, the average VaR almost grew 500% from the bottom during 2004-2005 to the peak in March 2009, and attain a level around $-20\%$ of return, which is almost twice larger than the peak in Chinese panel at the same period.

Figure 6: Average individual VaR of French panel and Chinese Panel.
Notes: The VaR is calculated at the 5% confidence level and the bootstrap is repeated 1000 times.
5.3 Overall systemic risk and Systemic importance

Figure 7 shows that the overall systemic risk\textsuperscript{12} (represented in black line and its level is on the right vertical axis), and its level was very low at the beginning of the crisis. Then the total systemic risk moved up significantly after the failure of Lehman Brother. After that, there has been a lot of panic in the market and the total systemic risk reached the peak around 140\% of loss in market capitalization return as the overall systemic contribution in March 2009. Since the release of US SCAP around early May 2009, the total systemic risk decrease quickly and returned to the pre-Lehman level. The market has calmed down till the first round of European sovereign debt crisis in May 2010, after the Greece receiving the aid with 14.5 billions euros, the total systemic risk decreased. Almost a year later, June 13 2011, Standard & Poor’s has downgraded Greek debt from B to CCC, the total systemic risk raised sharply and reached the peak in summer 2011 around 100\% of loss in return after 2007-2009 financial crisis.

![Figure 7: Marginal Contribution of Systemic Risk and Total Systemic Risk in French panel.](image)

On the left side of the graph, the axis represents the importance of systemic risk of each financial institution in percentage.\textsuperscript{13} The level of systemic importance during the whole sample is quite consistent, there is not much variation of weight of systemic importance among banks. BNP Paribas is the bank that have the most systemic

\textsuperscript{12} Delta-Multi CoVaR when all banks in the system are in financial distress, as formulated in equations (10) and (11).

\textsuperscript{13} This is achieved by the additivity of Shapley value.
important weight, Crédit Agricole and Société Générale are not too far from BNP Paribas. However, Natixis and Crédit Mutuel have the lowest systemic important weight in the system.

In table 3, I provide the summary statistics of the estimated Shapley-CoVaR series and Standard-CoVaR series for all banks in French panel. I first compute the mean of each series in different time horizon to have a general idea about the systemic importance among these banks, then I compute the standard deviation of each series to see the variation of the systemic importance for each bank. Note that the total risk in table 3 provides the benchmark for regulators to implement systemic risk measures individually. This benchmark gives the overall systemic risk contribution to the system when all banks are in distress. So the number in “mean column” of table 3 can be interpreted as, consider the number for BNP during the whole sample in Shapley-CoVaR case, BNP contributes, in average, -0.106 (with 5% confidence level) of return of market capitalization to the whole system; For standard-CoVaR measure, it contributes more than previous case as -0.285 of return of market capitalization to the whole system. Another interesting result shows in this table is that the more variation (high standard deviation) of the systemic risk series in time horizon comes with the more systemic importance of that series.

![Figure 8: Marginal Contribution of Systemic Risk and Total Systemic Risk in Chinese panel.](image)

Figure 8 represent the same information as in figure 7 for Chinese panel. Since the sample starts from the late 2006, the first peak for the total risk corresponds to the so called “Chinese correction” in early 2007. The second peak comes at the core
<table>
<thead>
<tr>
<th>Shapley-CoVaR</th>
<th>Std-CoVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole Sample</strong></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>Std</td>
</tr>
<tr>
<td>BNP</td>
<td>-0.106</td>
</tr>
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<td>SoGen</td>
<td>-0.096</td>
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<tr>
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<td>CrdMtl</td>
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<tr>
<td>Total Risk</td>
<td>-0.518</td>
</tr>
</tbody>
</table>

Table 3: Shapley-CoVaR and Standard CoVaR in French panel.

Notes: The whole sample period is from 4/19/2002 to 6/29/2012, the financial crisis period is from 6/24/2007 to 4/23/2010 and the European crisis period is from 5/1/2010 to 6/29/2012. The \( \alpha \)-scale of deviation to define the normal case is set \( \alpha = 0.5 \). Shapley-CoVaR refers to the methodology proposed in this paper, and Standard-CoVaR is proposed by Adrian and Brunnermeier (2011). Total risk refers to the systemic risk contribution when all these 5 banks are in distress, which is calculated by formulas (10) and (11). This is also the benchmark for regulators to implement systemic risk measures individually.

of 2007-2009 financial crisis, but Chinese banking system doesn’t suffer as much as French banking does, this is because the french banks have more connections with U.S. banks. This figure shows us that the most systemically important banks in China are the state banks for Industrial & Commercial Bank of China, China Construction Bank and bank of China. However, two private banks, Bank of Communications and China Merchants Bank have less systemic importance. In general, the magnitude of
systemic risk is less in China than in France. Table 4 reports the summary statistics of the estimated Shapley-CoVaR series and Standard-CoVaR series for all banks in Chinese panel.

<table>
<thead>
<tr>
<th>Shapley-CoVaR</th>
<th>Std-CoVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole Sample</strong></td>
<td><strong>Whole Sample</strong></td>
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<tr>
<td>Mean</td>
<td>Std</td>
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<td>ICBC</td>
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<td>CMBC</td>
<td>-0.053</td>
</tr>
<tr>
<td>Total Risk</td>
<td>-0.295</td>
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</tbody>
</table>

Table 4: Shapley-CoVaR and Standard CoVaR in French panel.

Notes: The whole sample period is from 11/3/2002 to 6/29/2012. The $\alpha$-scale of deviation to define the normal case is set $\alpha = 0.5$. Shapley-CoVaR refers to the methodology proposed in this paper, and Standard-CoVaR is proposes by Adrian and Brunnermeier (2011). Total risk refers to the systemic risk contribution when all these 5 banks are in distress, which is calculated by formulas (10) and (11). This is also the benchmark for regulators to implement systemic risk measures individually.

5.4 Additivity

Table 3 and 4 show an interesting result, the sum of individual $\Delta CoVaR$ is larger than the total systemic risk. This difference is quite small during the good period, this is because the correlations between banks are very small at that period. However, during the crisis period the correlations between banks increase sharply, and this implies that sum of individual $\Delta CoVaR$ is greater than the total systemic risk $\Delta CoVaR^{1\ldots N}$. If the central planner regulate the systemic risk based on standard $CoVaR$ that will punish the economy by over-regulating, since it will charge more than it should be. Furthermore, it will limit the volume of credit to the real economy, thus amplify the financial crisis and prolong the recovery of the economy. Note that the amount of over-regulation can be very significant, based on French system, the amount of over-regulation during the 2007-09 financial crisis and the European crisis are close to 100% of loss in return which is even larger than the total systemic risk at both periods around 50% of loss in return.

In the Shapley-CoVaR case of table 3 and 4, the sum of individuals’ average measures is exactly equal to the total risk. This is in line with Tarashev, Borio,
and Tsatsaronis (2010) and Drehmann and Tarashev (2011), based on Shapley value methodology, this systemic risk measure has a additive property, which is desirable from an operational perspective, since prudential requirements can be a linear transformation of the marginal contribution if the systemic risk measure is additive. Therefore, it allows the Macro-Prudential tools can be implemented at individual levels. Note that, the “Distressed Insurance Premium” and the “Marginal Expected Shortfall” also have additive property, but these measures are focusing on the exposure of individual institution when a systematic shock take place, they do not measuring the contagion (externality) that an institution contribute to the financial system. However, the systemic risk measure proposed in this paper captures the externality that a specific institutions contribute to the whole system efficiently.

6 Regulation

The macroprudential framework deals with systemic risk in both the time dimension (procyclicality) and the cross-sectional dimension (contagion). I have developed a systemic risk measure to deal with the latter, now, let’s focus on the former. Adrian and Shin (2011) shows that the procyclical leverage follows directly from the counter-cyclical nature of risk measure. To weaken the procyclical effect in the economy, one can change the characteristic of the proportion coefficient of the capital that intermediary holds per unit of risk measure. More specifically, I propose a counter-cyclical coefficient to attain this goal. Based on chapter 13 of Acharya and Richardson (2009) and the measure proposed above, the systemic capital charge (SCC) as a macroprudential tool could be written as:

$$SCC = \lambda \times Sh(v) \times \text{Value}$$

where $Sh(v)$ is the systemic risk contribution provide by multi-$\Delta CoVaR$ and Shapley value approach introduced in this paper.

This value can be market capitalization if one compute the measure by capitalization and it can be also the asset if the measure is derived by asset. The $Sh(v)$ is the systemic importance of systemic risk expressed in percent of value. However, how to choose the optimal counter-cyclical coefficient $\lambda$ is still not clear to regulators and

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14. Under the 1996 Market Risk Amendment of Basel capital accord, this coefficient is equal to three.
economists, this can be put into the agenda of the future research projects.

This macroprudential tool gives the incentives to financial institutions to limit their holding on systemic risk since capital reserves is costly, and countercyclical coefficient may weaken the procyclical effect in the economy.

7 Conclusion

In this paper I propose a new systemic risk measure to identify financial institutions’ systemic importance. I use \textit{Multi$-$CoVaR} to calculate the total systemic risk and then use the Shapley value methodology to efficiently allocate total systemic risk to each financial institution. The additivity property of the Shapley value ensures that the sum of each institution’s Shapley value of systemic risk contribution is exactly equal to the \textit{Multi$-$ΔCoVaR} of all the financial institutions in the system being in financial distress, hence the macroprudential policy can potentially be efficiently implemented based on this measure.

However, there are also some future research tasks on the field. First, on the strength of the methodology mentioned above, it is useful to construct a forward looking framework. With TGARCH and DCC models, it is possible to construct a forward looking conditional distribution, a forward looking individual risk is also feasible, since bootstrap technology could deliver this outcome. Second, what is the driving factors of systemic risk is needed, since if these driving factors can be identified, it would be very useful for financial institutions to control their systemic risk and make the financial market more stable. Third, how to design a countercyclical systemic capital charge can be also put into the agenda of the future research projects. Because this countercyclical systemic capital charge may weaken the procyclical effect in the economy.
References


Appendix

A Diagnostic of TGARCH model

I did a diagnosis of the TGARCH model to see whether these models are well specified or not. The objective of variance modeling is to construct a variance measure, which has the property that standardized squared returns, $r_i^2/\hat{\sigma}_i^2$ have no systematic autocorrelation.

We can see from graph that the standardized squared returns have no autocorrelation as the sticks of different lags are all in their standard error banks. Model is well specified in terms of in sample check.